Chapter 14
Web Text Box 2

The Goldman-Hodgkin-Katz equation

The Nernst equation (book page 234) allows us to calculate the equilibrium voltage for a particular ion under specific concentration conditions. It can be used to predict the membrane voltage of a cell, such as a glial cell, in which the plasma membrane is permeable to one ion only.

The plasma membranes of most cells are permeable to more than one ion, so that the membrane voltage settles down to a value at which the different ions are entering or leaving the cell, but the net current is zero so that overall the cytosol is neither gaining nor losing charge. The Goldman-Hodgkin-Katz equation (sometimes called the constant field equation) allows one to calculate the voltage at which the net current through passive pathways (that is, ion channels) is zero. The equation was derived in its standard form by Hodgkin and Katz (1949. Journal of Physiology, 108:37) using an earlier analysis by Goldman (1943. Journal of General Physiology, 27:37), hence the name of the equation. For a cell whose membrane is permeable to potassium and sodium the equation can be written

$$V_{i=0} = \left(\frac{RT}{F}\right) \log_e \left(\frac{P_K\,[K_{outside}] + P_{Na}\,[Na_{outside}]}{P_K\,[K_{inside}] + P_{Na}\,[Na_{inside}]}\right) \text{volts}$$

Where P_K is the permeability of the plasma membrane to potassium and P_{Na} is the permeability to sodium. As before, R is the gas constant (8.3 J mol^{-1} $degree^{-1}$), T is the absolute temperature and F, the number that relates the coulomb to the mole, has the value 96,500.

Although the equation has a similar form to the Nernst equation its derivation is very much more complicated. Notice that in the special case of a membrane permeable to one ion only, the equation simplifies to the Nernst equation. This is what we would expect, because when only one ion can carry current, the condition that no net current flows is the condition where that one ion does not move. However, it is important to remember that in the more general condition where the plasma membrane is permeable to more than one ion the individual ions <u>are</u> moving. At the voltage given by the Goldman-Hodgkin-Katz equation the cell is at a steady state because the net current is zero, but it is constantly gaining sodium and losing potassium, and has to expend energy to pump these ions back where they came from.

Sodium and potassium are pumped by the sodium/potassium ATPase . Since this moves three sodiums out for every two potassiums that move in, it generates its own current – we say that the ATPase is electrogenic. Thus if we want to calculate the voltage at which the cytosol is neither gaining nor losing charge, we should be calculating the voltage at which the total current, including that generated by the sodium/potassium ATPase, is zero. In 1963 Mullins and Noda derived an equation that allowed this to be calculated. For a cell whose membrane is permeable to potassium and sodium the equation can be written

$$V_{i=0} = \left(\frac{RT}{F}\right) \log_e \left(\frac{r\,P_K\,[K_{outside}] + P_{Na}\,[Na_{outside}]}{r\,P_K\,[K_{inside}] + P_{Na}\,[Na_{inside}]}\right) \text{volts}$$

where the new parameter r is the number of sodium ions moved out for every potassium ion moved in, that is, 1.5 (Mullins and Noda. 1963. Journal of General Physiology, 47:117).

Students approaching the question of how the resting voltage is generated sometimes fall into the trap of thinking that the cytosol is negative because the sodium/potassium ATPase is electrogenic. This is not the case, but nor is the effect of the current generated by the ATPase completely negligible. The difference between the voltage calculated by the Mullins and Noda equation and that given by the Goldman-Hodgkin-Katz equation is about 3 mV. Of course, this does not mean that the sodium/potassium

ATPase has only a small role in generating the resting voltage. It is only because the ATPase is constantly throwing out all the sodium leaking in, and pulling back all the potassium leaking out, that the concentration gradients that underlie the resting voltage are maintained.

The contribution of the ATPase current in a real cell can be measured easily by watching what happens when the ATPase is poisoned with a cardiac glycoside such as digitalis (Example 12.1 on book page 199) or ouabain. Immediately after application of the drug the membrane voltage shifts a few mV positive, representing the difference between the voltage predicted by the Mullins-Noda equation and the Goldman-Hodgkin-Katz voltage. Then over the subsequent minutes the membrane voltage slowly decays to zero as the concentration gradients run down.